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2003 J. Phys.: Condens. Matter 15 2797

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INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **15** (2003) 2797–2809 PII: S0953-8984(03)56158-6

Magnon–paramagnon effective theory of itinerant ferromagnets

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Received 15 November 2002
Published 22 April 2003
Online at stacks.iop.org/JPhysCM/15/2797

Abstract
The present work is devoted to the derivation of an effective magnon–paramagnon theory starting from a microscopic single-band lattice model of ferromagnetic metals. For some values of the microscopic parameters it reproduces the Heisenberg theory of localized spins. For small magnetization the effective model describes the physics of weak ferromagnets. It allows us to account for the magnon–magnon and magnon–paramagnon interactions going beyond Moriya's theory. The effective theory is written in a way which keeps $O(3)$ symmetry manifest, and describes both the ordered and disordered phases of the system.

To derive the effective model a Schwinger-bosons–slave-fermions representation of the operators is used. Within this approach the local Coulomb repulsion is treated exactly, and as a result, the constants in the effective theory are finite and well defined for all values of the magnetization.

An equation for the Curie temperature, which takes the magnon fluctuations into account exactly, is obtained. For weak ferromagnets, in the spin-wave approximation, the critical temperature scales like $T_c \sim m^{5/3}$. It is well below the Stoner critical temperature $T_c \sim m$ and the critical temperature obtained within Moriya's theory $T_c \sim m^{3/2}$.

1. Introduction

The Heisenberg model of ferromagnetism, based on the exchange interaction of localized electrons, gives an explanation of many properties of non-conducting magnetic systems at both low and high temperatures. However, the development of a satisfactory theory of ferromagnetic metals has run into difficulties.

Theories of weak ferromagnetic metals based on the Landau Fermi liquid theory have been developed by several theorists [1–4]. The spectrum of the spin excitations has been found. It consists of spin fluctuations of paramagnon type and a transverse spin-wave branch.

Murata and Doniach [5] have proposed a phenomenological mode–mode coupling theory to describe the temperature dependence of the quantities for a weak ferromagnet. However, they started from a classical Hamiltonian and ignored the quantum effects, which are important because of the low temperature nature of the weak ferromagnet.

An alternative approach to magnetic phase transitions in Fermi systems has been developed by Moriya and Kawabata [6, 7]. It is a self-consistent one-loop approximation which accounts for the spin fluctuations.

The nonlinear effects of spin fluctuations are treated in [8], using a self-consistent rotationally invariant Hartree approximation.

New results concerning magnetic phase diagrams in Hubbard-type models have been obtained within the dynamical mean-field theory [9, 10].

Perhaps the most striking feature of the itinerant ferromagnets is the quantum transition to paramagnet. In [11], Hertz derives an effective paramagnon theory of a paramagnet to ferromagnet quantum phase transition. He analysed the effective model by means of a renormalization group method that generalizes Wilson's treatment of classical phase transition and concluded that the critical behaviour of itinerant ferromagnets in dimensions larger than one are described by a mean-field fixed point. Hertz's work has been reexamined in [12]. The results of this paper support those of Hertz in the 3D case but in many aspects differ from them in 2D.

A more realistic model of ferromagnetic metals is considered in [13, 14]. It contains a particle–hole spin-triplet interaction that causes ferromagnetism, as well as particle–hole spin-singlet and particle–particle interactions. The spin-triplet interaction is decoupled by introducing a vector field whose average is proportional to the magnetization, and by performing a Hubbard–Stratanovich transformation. Then the fermionic degrees of freedom are integrated out accounting for the rest part of interaction by means of perturbation theory. The resulting effective field theory is nonlocal. It contains an effective long-range interaction between the order parameter fluctuations. The analysis in [13, 14] is restricted to power counting arguments at tree level. It shows that the critical behaviour is governed by a Gaussian fixed point and that all non-Gaussian terms are irrelevant in the renormalization group sense. Logarithmic corrections to power-law scaling are obtained in the $D = 3$ case. For $D < 3$ the deviations from mean-field theory results depend on D.

There is a considerable interest in the finite temperature properties of weak itinerant ferromagnets. Experimentally, the transition in the weak ferromagnet MnSi has been investigated at different Curie temperatures by applying hydrostatic pressure [15]. The transition at high temperatures was found to be of second order, while at lower transition temperatures it is of first order. In the first-order regime the transition temperature was found to scale with pressure as $T_c \sim (p_c - p)^{1/2}$. The scaling law is explained by a mean-field analysis, assuming a dynamical exponent $z = 3$. But in Hertz's theory [11], the dynamical critical exponent is equal to three due to paramagnon excitations. Hence, in order to explain correctly the characteristic features of itinerant ferromagnets it should be important to work with an effective theory that keeps explicitly the magnon as well as paramagnon excitations.

Many decades of research on magnetism have led to the view that the disordered state of itinerant ferromagnetism could be described in terms of Fermi liquid theory. Very recent experiments [16] showed that this might be wrong. The experiments using MnSi reveal that the Curie temperature falls monotonically with pressure p. The transition is second order up to $p^* = 12$ kbar, where $T_c = 12$ K. The transition is, however, weakly first order between p^* and p_c where T_c falls to zero. The main result is that in the interval $p^* < p < p_c$, and above T_c, the resistivity exhibits a temperature dependence of the form $\rho(T) = \rho_0 + AT^{3/2}$, showing the non-Fermi liquid nature of the normal state. The conventional T^2 form of resistivity is observed in the ferromagnetic state and the normal state at pressures below p^* and above p_c. In the ferromagnetic phase the spin fluctuations (magnons and paramagnons) renormalize the fermion's parameters [17], keeping the Fermi liquid nature unchanged. When the system undergoes a second-order phase transition, the magnons open a gap and the paramagnetic Fermi liquid state is the ground state of the system. The $T^{3/2}$ power law of the resistivity is not consistent with current models of itinerant ferromagnetism [4, 6–8, 10]. The origin of this form of $\rho(T)$ may lie in extremely strong spin fluctuations near the quantum ferromagnetic to paramagnetic transition, which changes the nature of the fermions' ground state. To understand this striking phenomena one needs an effective model of spin–spin interactions which describes both the ordered and disordered phases of the system.

The present work is devoted to the derivation of an effective magnon–paramagnon theory starting from a microscopic, single-band lattice model of ferromagnetic metals. This is an effective theory of ferromagnets, and for some values of microscopic parameters it reproduces the Heisenberg theory of localized spins. On the other hand, for small magnetization the effective model describes the physics of weak ferromagnets. Thus, the effective magnon–paramagnon theory interpolates between theories of localized and itinerant electrons. The zero-temperature dimensionless magnetization per lattice site m is introduced to describe this interpolation. It parametrizes the ground state of the system, and moving m we change the ground state and respectively the fluctuations above it. When m is maximal ($m = \frac{1}{2}$) all the lattice sites are occupied by one electron with spin up in the ground state, and the only relevant excitations are magnons. The effective theory is the Heisenberg theory of localized spins. When the magnetization is smaller, i.e. some of the lattice sites are doubly occupied or empty in the ground state, the spectrum consists of paramagnon and magnon excitations and the effective theory is a 'spin m' Heisenberg theory coupled to paramagnon fluctuations. Decreasing the magnetization results in changing the parameters of the spin fluctuations, and close to the quantum critical point $m = 0$ the paramagnon becomes important due to the singularity in the paramagnon propagator. In the quantum paramagnetic phase ($m = 0$), the magnon excitations disappear from the spin spectrum and one obtains Hertz's effective model.

The ferromagnetic order parameter is a vector $\vec{M}$ field. The transverse spin fluctuations (magnons) are described by $M_1 + \mathrm{i}M_2(M_1 - \mathrm{i}M_2)$ and the longitudinal fluctuations (paramagnons) by $M_3 - \langle M_3\rangle$. Alternatively the vector field can be written as a product of its amplitude $\rho = \sqrt{M_1^2 + M_2^2 + M_3^2}$ and an unit vector $\vec{n}$, $\vec{M} = \rho\vec{n}$. In the ferromagnetic phase one sets $M_3 = \langle M_3\rangle + \varphi$ and in the linear (spin-wave) approximation one obtains $\rho = \langle M_3\rangle + \varphi$. It is evident now that the fluctuations of the ρ field, $\rho - \langle M_3\rangle$, are exactly the paramagnon excitations in a formalism which keeps $O(3)$ symmetry manifest. One can write the effective theory in terms of $\vec{M}$-vector components or, equivalently, in terms of ρ and an unit vector $\vec{n}$. I use the parametrization in terms of unit vector and spin singlet amplitude because the unit vector $\vec{n}$ describes the true Goldstone modes of the order parameter. The effective action keeps the $O(3)$ symmetry manifest and describes both the ordered and disordered phases of the system. Above the Curie temperature the spectrum consists of spin-singlet fluctuations of paramagnon type and spin-$\frac{1}{2}$ spinon fluctuations. The spinon has a gap, but near the critical temperature it approaches zero and spin-$\frac{1}{2}$ fluctuations as well as paramagnons are essential in describing the thermal phase transition of itinerant ferromagnets.

An equation for the Curie temperature, which takes the magnon fluctuations into account exactly, is obtained. For weak ferromagnets the critical temperature scales like $T_c \sim m^{5/3}$. It is well below the Stoner critical temperature $T_c \sim m$ and the critical temperature obtained within Moriya's theory $T_c \sim m^{3/2}$ [18].

Scaling arguments based on Hertzs renormalization group technique allow us to obtain a relation between spin stiffness and the inverse longitudinal magnetic susceptibility. The relation is a consequence of the fact that the quantum critical exponent is equal to 3. Making use of the relation one can obtain the contribution of magnon scattering to the inverse longitudinal magnetic susceptibility.

To derive the effective model a Schwinger-bosons–slave-fermions representation of the operators is used. The salient point is that within this approach the local Coulomb repulsion is treated exactly. As a result, the constants in the effective theory are finite and well defined for all values of the magnetization m as opposed to the nonlocal effective theory (see [13, 14]), where the Coulomb interaction is accounted for perturbatively.

The advantage of the single-band model is its comparative mathematical simplicity. It is simple enough to handle in detail, but yet close enough to physical realty to supply with useful information, and the obtained effective model to be of general application. It is physically motivated to discuss this model if the Fermi surface lies within a single conduction band, and if this band is well separated from the other bands and the interaction is not too strong [19].

The paper is organized as follows. In section 2 an effective magnon–paramagnon theory of ferromagnetic metals is obtained. The Curie temperature is calculated and written in terms of magnetization and spin stiffness constants. In section 3, Hertz's renormalization group in tree approximation is extended to include the magnon fluctuations. Section 4 is devoted to the concluding remarks.

2. Magnon–paramagnon effective model

The simplest, single-band lattice model of ferromagnetic metals is determined by the Hamiltonian [20, 21]

$$\hat{H} = -t \sum_{\langle i,j\rangle,\sigma} (\hat{c}^{+}_{i\sigma}\hat{c}_{j\sigma} + \text{h.c.}) - J \sum_{\langle i,j\rangle} \hat{\vec{S}}_i \cdot \hat{\vec{S}}_j + U \sum_i \hat{n}_{i\uparrow}\hat{n}_{i\downarrow} - \mu \sum_i \hat{n}_i. \tag{1}$$

Here $\hat{c}^{+}_{i\sigma}$ and $\hat{c}_{j\sigma}$ are creation and annihilation operators for electrons, $\hat{n}_{i\sigma} = \hat{c}^{+}_{i\sigma}\hat{c}_{i\sigma}$ and $\hat{n}_i = \sum_\sigma \hat{n}_{i\sigma}$ are density operators, and $\hat{\vec{S}}_i = \frac{1}{2}\sum_{\sigma\sigma'} \hat{c}^{+}_{i\sigma}\vec{\tau}_{\sigma\sigma'}\hat{c}_{i\sigma'}$, where $\vec{\tau}$ denotes the vector of Pauli matrices, are spin operators. The sums are over all sites of a three-dimensional cubic lattice, $\langle i, j\rangle$ denotes the sum over the nearest neighbours and μ is the chemical potential. In (1) the parameter J is an off-diagonal matrix element of the Coulomb interaction between electrons in Wannier states at nearest-neighbour sites which is generically ferromagnetic ($J > 0$) in nature and U is the usual Hubbard on-site repulsion [19].

In terms of Schwinger bosons ($\hat{\varphi}_{i,\sigma}, \hat{\varphi}^{\dagger}_{i,\sigma}$) and slave fermions ($\hat{h}_i, \hat{h}^{\dagger}_i, \hat{d}_i, \hat{d}^{\dagger}_i$) the operators have the following representation:

$$\begin{aligned} &\hat{c}_{i\uparrow} = \hat{h}^{\dagger}_i\hat{\varphi}_{i1} + \hat{\varphi}^{\dagger}_{i2}\hat{d}_i, \qquad \hat{c}_{i\downarrow} = \hat{h}^{\dagger}_i\hat{\varphi}_{i2} - \hat{\varphi}^{\dagger}_{i1}\hat{d}_i, \qquad \hat{n}_i = 1 - \hat{h}^{+}_i\hat{h}_i + \hat{d}^{+}_i\hat{d}_i, \\ &\hat{c}^{\dagger}_{i\uparrow}\hat{c}_{i\uparrow}\hat{c}^{\dagger}_{i\downarrow}\hat{c}_{i\downarrow} = \hat{d}^{\dagger}_i\hat{d}_i, \qquad \hat{\vec{S}}_i = \frac{1}{2}\sum_{\sigma\sigma'} \hat{\varphi}^{+}_{i\sigma}\vec{\tau}_{\sigma\sigma'}\hat{\varphi}_{i\sigma'}, \qquad \hat{\varphi}^{\dagger}_{i\sigma}\hat{\varphi}_{i\sigma} + \hat{d}^{\dagger}_i\hat{d}_i + \hat{h}^{\dagger}_i\hat{h}_i = 1. \end{aligned} \tag{2}$$

The partition function can be written as a path integral over the complex functions of the Matsubara time τ $\varphi_{i\sigma}(\tau)(\bar{\varphi}_{i\sigma}(\tau))$ and Grassmann functions $h_i(\tau)(\bar{h}_i(\tau))$ and $d_i(\tau)(\bar{d}_i(\tau))$ [22]:

$$\mathcal{Z}(\beta) = \int D\mu\,(\bar{\varphi}, \varphi, \bar{h}, h, \bar{d}, d,)\mathrm{e}^{-S}. \tag{3}$$

The action is given by the expression

$$S = \int_0^\beta \mathrm{d}\tau \left[\sum_i (\bar{\varphi}_{i\sigma}(\tau)\dot{\varphi}_{i\sigma}(\tau) + \bar{h}_i(\tau)\dot{h}_i(\tau) + \bar{d}_i(\tau)\dot{d}_i(\tau)) + H(\bar{\varphi}, \varphi, \bar{h}, h, \bar{d}, d)\right], \tag{4}$$

where β is the inverse temperature and the Hamiltonian is obtained from equations (1) and (2) replacing the operators with the functions. In terms of Schwinger bosons and slave fermions the theory is $U(1)$ gauge-invariant, and the measure includes δ functions that enforce the constraint and the gauge-fixing condition

$$D\mu(\bar{\varphi}, \varphi, \bar{h}, h, \bar{d}, d) = \prod_{i,\tau,\sigma} \frac{D\bar{\varphi}_{i\sigma}(\tau) D\varphi_{i\sigma}(\tau)}{2\pi i} \prod_{i\tau} D\bar{h}_i(\tau) Dh_i(\tau) D\bar{d}_i(\tau) Dd_i(\tau)$$
$$\times \prod_{i\tau} \delta(\bar{\varphi}_{i\sigma}(\tau)\varphi_{i\sigma}(\tau) + \bar{h}_i(\tau)h_i(\tau) + \bar{d}_i(\tau)d_i(\tau) - 1) \prod_{i\tau} \delta(g.f). \tag{5}$$

I make a change of variables, introducing new Bose fields $f_{i\sigma}(\tau)(\bar{f}_{i\sigma}(\tau))$ [23]:

$$\begin{aligned} f_{i\sigma}(\tau) &= \varphi_{i\sigma}(\tau)(1 - \bar{h}_i(\tau)h_i(\tau) - \bar{d}_i(\tau)d_i(\tau))^{-1/2}, \\ \bar{f}_{i\sigma}(\tau) &= \bar{\varphi}_{i\sigma}(\tau)(1 - \bar{h}_i(\tau)h_i(\tau) - \bar{d}_i(\tau)d_i(\tau))^{-1/2}, \end{aligned} \tag{6}$$

where the new fields satisfy the constraint

$$\bar{f}_{i\sigma}(\tau) f_{i\sigma}(\tau) = 1. \tag{7}$$

In terms of the new fields the spin vector and the action have the form

$$\vec{S}_i(\tau) = \frac{1}{2} \sum_{\sigma\sigma'} f^+_{i\sigma}(\tau) \vec{\tau}_{\sigma\sigma'} f_{i\sigma'}(\tau)(1 - \bar{h}_i(\tau)h_i(\tau) - \bar{d}_i(\tau)d_i(\tau)) \tag{8}$$

$$S = \int_0^\beta \mathrm{d}\tau \left\{ \sum_i \left[\bar{f}_{i\sigma}(\tau)\dot{f}_{i\sigma}(\tau) + \bar{h}_i(\tau)\left(\frac{\partial}{\partial\tau} - \bar{f}_{i\sigma}(\tau)\dot{f}_{i\sigma}(\tau)\right)h_i(\tau) \right.\right.$$
$$\left.\left. + \bar{d}_i(\tau)\left(\frac{\partial}{\partial\tau} - \bar{f}_{i\sigma}(\tau)\dot{f}_{i\sigma}(\tau)\right)d_i(\tau)\right] + H(\bar{f}, f, \bar{h}, h, \bar{d}, d)\right\}, \tag{9}$$

where $H(\bar{f}, f, \bar{h}, h, \bar{d}, d)$ is the Hamiltonian

$$H = -t \sum_{\langle i,j\rangle} [\bar{d}_i d_j \bar{f}_{j\sigma} f_{i\sigma} + \bar{d}_j d_i \bar{f}_{i\sigma} f_{j\sigma} - \bar{h}_i h_j \bar{f}_{j\sigma} f_{i\sigma} - \bar{h}_j h_i \bar{f}_{i\sigma} f_{j\sigma}$$
$$+ (h_i d_j - h_j d_i)(\bar{f}_{i1}\bar{f}_{j2} - \bar{f}_{i2}\bar{f}_{j1}) + (\bar{d}_j\bar{h}_i - \bar{d}_i\bar{h}_j)(f_{i1}f_{j2} - f_{i2}f_{j1})]$$
$$\times (1 - \bar{h}_i h_i - \bar{d}_i d_i)^{1/2}(1 - \bar{h}_j h_j - \bar{d}_j d_j)^{1/2}$$
$$+ \frac{J}{2}\sum_{\langle i,j\rangle}[\bar{h}_i h_i + \bar{d}_i d_i + \bar{h}_j h_j + \bar{d}_j d_j] - \frac{J}{2}\sum_{\langle i,j\rangle}(\bar{h}_i h_i + \bar{d}_i d_i)(\bar{h}_j h_j + \bar{d}_j d_j)$$
$$+ \frac{J}{8}\sum_{\langle i,j\rangle}(\vec{n}_j - \vec{n}_i)^2(1 - \bar{h}_i h_i - \bar{d}_i d_i)^{1/2}(1 - \bar{h}_j h_j - \bar{d}_j d_j)^{1/2}$$
$$+ U\sum_i \bar{d}_i d_i - \mu \sum_i (1 - \bar{h}_i h_i + \bar{d}_i d_i). \tag{10}$$

In equation (10) $\vec{n}_i = \sum_{\sigma\sigma'} \bar{f}_{i\sigma}\vec{\tau}_{\sigma,\sigma'} f_{i\sigma'}$ is a unit vector. Equation (8) describes in an $O(3)$ covariant way the spin. When the lattice site is empty or doubly occupied the spin vector is zero. When the lattice site is occupied by one electron the unit vector $\vec{n}_i$ identifies the local orientation. One can consider the first two components n_{i1} and n_{i2} as independent, and then $n_{i3} = \sqrt{1 - n_{i1}^2 - n_{i2}^2}$. In the leading order of the fields, the spin vector has the form

$$S_{i1} \simeq \tfrac{1}{2} n_{i1}, \qquad S_{i2} \simeq \tfrac{1}{2} n_{i2}, \qquad S_{i3} - \tfrac{1}{2} \simeq -\tfrac{1}{2}(\bar{h}_i h_i + \bar{d}_i d_i). \tag{11}$$

The last equation shows that the longitudinal spin fluctuations are associated with the collective fields $(\bar{h}_i h_i + \bar{d}_i d_i)$.

To avoid misunderstandings, it is important to point out that the charge waves are associated with the collective field $(\bar{d}_i d_i - \bar{h}_i h_i)$ (see the representation of the electron number operator (2)).

To formulate a mean-field theory I drop the terms of order equal or higher than six in the Hamiltonian equation (10). It is convenient to replace the term $\frac{J}{2}\sum_{\langle i,j\rangle}(\bar{h}_i h_i + \bar{d}_i d_i)(\bar{h}_j h_j + \bar{d}_j d_j)$ in equation (10) with the local one $\frac{3J}{2}\sum_i(\bar{h}_i h_i + \bar{d}_i d_i)^2$. The difference is in higher order of derivatives and I will drop it. Then one can decouple this term, by means of the Hubbard–Stratanovich transformation, introducing a real, spin-singlet and gauge-invariant field:

$$\exp\left(\frac{3J}{2}\int_0^\beta \mathrm{d}\tau \sum_i(\bar{h}_i(\tau)h_i(\tau)+\bar{d}_i(\tau)d_i(\tau))^2\right) = \int\prod_{i\tau} DS_i(\tau) \times \exp\left(-\int_0^\beta \mathrm{d}\tau \sum_i\left[\frac{3J}{2}S_i(\tau)S_i(\tau) - 3J(\bar{h}_i(\tau)h_i(\tau)+\bar{d}_i(\tau)d_i(\tau))S_i(\tau)\right]\right). \tag{12}$$

Now, the action is quadratic with respect to the fermions and one can integrate them out. The resulting action depends on the spinons and the real field S_i. It has a minimum at the point $S_i = s_0$, $f_{i\sigma} = f_\sigma$ and the stationary condition is

$$s_0 = \langle \bar{h}_i h_i + \bar{d}_i d_i\rangle. \tag{13}$$

Expanding the effective action around this point one obtains the effective model.

To improve the calculations I account for terms of order six and higher, replacing the collective field $\bar{h}_i h_i + \bar{d}_i d_i$ in these terms by its mean-field value from equation (13). The new Hamiltonian depends on the fields $\bar{f}_{i\sigma}(\tau)$, $f_{i\sigma}(\tau)$, $2\varphi_i(\tau) = s_0 - S_i(\tau)$, and is quadratic with respect to the fermions

$$\begin{aligned} H = {} & -t\sum_{\langle i,j\rangle}[\bar{d}_i d_j + \bar{d}_j d_i - \bar{h}_i h_j - \bar{h}_j h_i] + (6mJ + U - \mu)\sum_i \bar{d}_i d_i + (6mJ+\mu)\sum_i \bar{h}_i h_i \\ & - 2mt\sum_{\langle i,j\rangle}[(\bar{d}_i d_j - \bar{h}_i h_j)(\bar{f}_{j\sigma} f_{i\sigma} - 1) + (\bar{d}_j d_i - \bar{h}_j h_i)(\bar{f}_{i\sigma} f_{j\sigma} - 1) \\ & + (h_i d_j - h_j d_i)(\bar{f}_{i1}\bar{f}_{j2} - \bar{f}_{i2}\bar{f}_{j1}) + (\bar{d}_j \bar{h}_i - \bar{d}_i \bar{h}_j)(f_{i1} f_{j2} - f_{i2} f_{j1})] \\ & + 6J\sum_i \varphi_i(\bar{h}_i h_i + \bar{d}_i d_i) + \frac{(2m)^2 J}{8}\sum_{\langle i,j\rangle}(\vec{n}_j - \vec{n}_i)^2 \end{aligned} \tag{14}$$

where $m = \frac{1}{2}(1 - s_0)$. Integrating out the fermions one obtains the action of the effective theory.

To get an intuition how the effective action looks, it is important to stress that the spinon fields contribute the action through the fields $\bar{f}_{i\sigma}(\tau)\dot{f}_{i\sigma}(\tau)$, $\bar{f}_{i\sigma}(\tau)(f_{j\sigma}(\tau) - f_{i\sigma}(\tau))$, $(f_{i1}(\tau)f_{j2}(\tau) - f_{j1}(\tau)f_{i2}(\tau))$ and $(\vec{n}_j - \vec{n}_i)^2$ (see equation (14)). In the continuum limit they have the form $\bar{f}_\sigma \partial_\mu f_\sigma(\mu = \tau, x, y, z)$, $(f_1\partial_\nu f_2 - f_2\partial_\nu f_1)$ $(\nu = x, y, z)$ and $\partial_\nu\vec{n}\cdot\partial_\nu\vec{n}$. The 4-vector $A_\mu = i\bar{f}_\sigma\partial_\mu f_\sigma$ transforms as an $U(1)$ gauge field, $(f_1\partial_\nu f_2 - f_2\partial_\nu f_1)$ as a charge-two complex field and $\partial_\nu\vec{n}\cdot\partial_\nu\vec{n}$ is gauge invariant. Hence, the simplest gauge-invariant and spin-singlet contributions of the first two fields have the form $(\bar{f}_1\partial_\nu\bar{f}_2 - \bar{f}_2\partial_\nu\bar{f}_1)(f_1\partial_\nu f_2 - f_2\partial_\nu f_1)$ and $(\partial_{\mu_1}A_{\mu_2} - \partial_{\mu_2}A_{\mu_1})(\partial_{\mu_1}A_{\mu_2} - \partial_{\mu_2}A_{\mu_1})$. It is not difficult to check that

$$(\bar{f}_1\partial_\nu\bar{f}_2 - \bar{f}_2\partial_\nu\bar{f}_1)(f_1\partial_\nu f_2 - f_2\partial_\nu f_1) = \tfrac{1}{4}\partial_\nu\vec{n}\cdot\partial_\nu\vec{n}. \tag{15}$$

The term $(\partial_{\mu_1}A_{\mu_2} - \partial_{\mu_2}A_{\mu_1})(\partial_{\mu_1}A_{\mu_2} - \partial_{\mu_2}A_{\mu_1})$ is of the same order as $(\partial_\nu\vec{n}\cdot\partial_\nu\vec{n})^2$ and I will ignore it. To obtain the effective theory it is convenient to set the gauge field $A_\mu = i\bar{f}_\sigma\partial_\mu f_\sigma$ equal to zero, and to account for the contribution of the complex field $(f_1\partial_\nu f_2 - f_2\partial_\nu f_1)$, the real field is φ and $\partial_\nu\vec{n}\cdot\partial_\nu\vec{n}$. An important exclusion is the $\bar{f}_\sigma\partial_\tau f_\sigma$ field which contributes linearly to the action.

Expanding the effective functional around the mean-field point and keeping only the first three terms, one can write the effective action in the form

$$S_{eff} = S_H + S_p + S_{int}. \tag{16}$$

S_H is the action of the Heisenberg theory of localized spins. In continuum limit it has the form

$$S_H = \int \mathrm{d}\tau\, \mathrm{d}^3\vec{r} \left[2m\bar{f}_\sigma(\tau,\vec{r})\dot{f}_\sigma(\tau,\vec{r}) + \frac{m^2 J_r}{2}\sum_{\nu=1}^{3}\partial_\nu\vec{n}(\tau,\vec{r})\cdot\partial_\nu\vec{n}(\tau,\vec{r})\right]. \tag{17}$$

In equation (17), $m = \frac{1}{2}(1-s_0)$, and s_0 comes from 'tadpole' diagrams with one h or d line, where the h and d dispersions are

$$\begin{aligned}\epsilon_k^h &= 2t(\cos k_x + \cos k_y + \cos k_z) + 6Jm + \mu,\\ \epsilon_k^d &= -2t(\cos k_x + \cos k_y + \cos k_z) + 6Jm - \mu + U.\end{aligned} \tag{18}$$

The renormalized exchange coupling constant has the following representation in terms of microscopic parameters and at zero temperature[1]

$$J_r = J - \frac{4t^2}{12Jm+U} + \frac{8}{3}\frac{t^2}{12Jm+U}\frac{1}{N}\sum_k\left(\sum_{\nu=1}^{3}\sin^2 k_\nu\right)(n_k^h + n_k^d). \tag{19}$$

In equation (19), n_k^d and n_k^h are the occupation numbers for d and h fermions, respectively. The first term is due to the direct Heisenberg exchange term in equation (14), and the other terms are due to Anderson's superexchange. The last two terms are obtained by calculating the one-loop self-energy diagrams of h and d fermions. The superexchange contribution to the exchange coupling constant goes to zero for small magnetization. Hence, near the quantum phase transition one can replace the renormalized coupling constant J_r by a bare one J.

S_p is the contribution to the effective action of the paramagnon excitations

$$S_p = \frac{1}{2}\int\frac{\mathrm{d}\omega}{2\pi}\frac{\mathrm{d}^3p}{(2\pi)^3}\varphi(\omega,\vec{p})\left(r + a\frac{|\omega|}{p} + bp^2\right)\varphi(-\omega,-\vec{p}) \tag{20}$$

where

$$r = 12J[1 - 3J(N(\epsilon_F^h) + N(\epsilon_F^d))] \tag{21}$$

and the constants a and b are calculated in a continuum limit:

$$a = 18\pi J^2\left(\frac{N(\epsilon_F^h)}{v_F^h} + \frac{N(\epsilon_F^d)}{v_F^d}\right) \qquad b = 3J^2\left(\frac{N(\epsilon_F^h)}{(k_F^h)^2} + \frac{N(\epsilon_F^d)}{(k_F^d)^2}\right). \tag{22}$$

It is obtained from the Lindhard functions for h and d fermions in the limit when p and $\frac{\omega}{p}$ are small.

Finally, the spinon–paramagnon interaction has the form

$$S_{int} = m^2\lambda\int\mathrm{d}\tau\,\mathrm{d}^3\vec{r}\,\varphi(\tau,\vec{r})\left[\sum_{\nu=1}^{3}\partial_\nu\vec{n}(\tau,\vec{r})\cdot\partial_\nu\vec{n}(\tau,\vec{r})\right] \tag{23}$$

where

$$\lambda = \frac{16Jt^2}{(12Jm+U)^2}\frac{1}{N}\sum_k\left(\sum_{\nu=1}^{3}\sin^2 k_\nu\right)(1 - n_k^h - n_k^d). \tag{24}$$

[1] The present discussion is correct if the renormalized exchange coupling constant is positive. To this end, it is sufficient for the microscopic parameters to satisfy the inequality $4t^2/U < J$.

The effective magnon–paramagnon coupling is obtained from triangular diagrams with two h and one d lines or with two d and one h lines.

To analyse the effective model, it is more convenient to rewrite it in terms of rescaled spinon fields

$$\bar{\zeta}_{i\sigma} = \sqrt{2m}\,\bar{f}_{i\sigma}, \qquad \zeta_{i\sigma} = \sqrt{2m}\,f_{i\sigma}. \tag{25}$$

The new fields satisfy the constraint

$$\bar{\zeta}_{i\sigma}\zeta_{i\sigma} = 2m, \tag{26}$$

and the action of the effective theory has the form

$$S_{eff} = \int \mathrm{d}\tau\, \mathrm{d}^3\vec{r} \left[\bar{\zeta}_\sigma(\tau,\vec{r})\dot{\zeta}_\sigma(\tau,\vec{r}) + \frac{J_r}{2}\sum_{\nu=1}^{3} \partial_\nu \vec{M}(\tau,\vec{r})\cdot\partial_\nu\vec{M}(\tau,\vec{r}) + \frac{\lambda}{4}\varphi(\tau,\vec{r})\left[\sum_{\nu=1}^{3}\partial_\nu\vec{M}(\tau,\vec{r})\cdot\partial_\nu\vec{M}(\tau,\vec{r})\right]\right] + S_p, \tag{27}$$

where $\vec{M}$ is the spin vector

$$\vec{M} = \tfrac{1}{2}\bar{\zeta}_\sigma\,\vec{\tau}_{\sigma,\sigma'}\zeta_{\sigma'}, \qquad \vec{M}^2 = m^2 \tag{28}$$

and S_p is given by equation (20).

It follows from equation (8) that the dimensionless magnetization of the system, per lattice site is defined by the equation

$$\langle S_i^3\rangle = \tfrac{1}{2}\langle n_i^3\rangle(1 - \langle \bar{h}_i h_i + \bar{d}_i d_i\rangle). \tag{29}$$

At zero temperature $\langle n_i^3\rangle = 1$ and using equation (13) one obtains that m is the zero-temperature dimensionless magnetization of the system per lattice site, $m = \langle S_i^3\rangle$. The parameter m depends on the microscopic parameters of the theory and characterizes the vacuum. If, in the vacuum state, every lattice site is occupied by one electron with spin up, then $m = \frac{1}{2}$ ($s_0 = 0$), the parameters a and b from equation (20) are equal to zero and $r = \frac{3J}{2}$. In this case one can integrate over the paramagnons and the resulting theory is the spin $\frac{1}{2}$ Heisenberg theory of the localized spins. When, in the vacuum state, some of the sites are doubly occupied ($\langle \bar{d}_i d_i\rangle \neq 0$) or empty ($\langle \bar{h}_i h_i\rangle \neq 0$), then $m < \frac{1}{2}$, the relevant excitations are the spinon and paramagnon excitations and the effective theory is a 'spin m' Heisenberg theory coupled to paramagnon fluctuations defined by equations (26)–(28). The system approaches the quantum critical point when $m \to 0$ ($s_0 \to 1$). One can see directly, from the stationary condition (13), that $r(m)$ approaches zero when $m \to 0$. Hence, the parameter r measures the distance from the quantum critical point. In a quantum paramagnetic phase ($m = 0$), the spinon excitations disappear from the spin spectrum (see equations (26) and (28)) and one obtains Hertz's effective model. One can add a four-paramagnon term, calculating one-loop diagrams with four h or d fermion lines, but I have dropped it, motivated by Hertz's result.

The effective theory is $U(1)$ gauge-invariant. Below the Curie temperature it is convenient to introduce explicitly the magnon excitations. To this end, I impose the gauge-fixing condition in the form $\arg\zeta_{i1} = 0$. Then the constraint (26) can be solved by means of the complex field $a_i(\tau) = \zeta_{i2}$ and $\zeta_{i1} = \sqrt{2m - \bar{a}_i(\tau)a_i(\tau)}$. For the components of the spin vector $M^+ = M_1 + \mathrm{i}M_2$, $M^- = M_1 - \mathrm{i}M_2$, and M_3 one obtains the Holstein–Primakoff representation:

$$M_i^+(\tau) = \sqrt{2m - \bar{a}_i(\tau)a_i(\tau)}\,a_i(\tau), \qquad M_i^-(\tau) = \bar{a}_i(\tau)\sqrt{2m - \bar{a}_i(\tau)a_i(\tau)}, \qquad M_i^3(\tau) = m - \bar{a}_i(\tau)a_i(\tau). \tag{30}$$

The kinetic term in the action and the measure are the same as the kinetic term and the measure in the theory of Bose field. The only difference is that the complex fields are subject to the condition $\bar{a}_i(\tau)a_i(\tau) \leqslant 1$.

In the spin-wave theory one approximates $\sqrt{2m - \bar{a}_i(\tau)a_i(\tau)}$ and integrates over the whole complex plane. Then, the model is simplified and can be written in terms of magnon $a_i(\tau)(\bar{a}_i(\tau))$ and paramagnon $\varphi_i(\tau)$ fields:

$$S_{eff} = \int \frac{d\omega}{2\pi} \frac{d^3 p}{(2\pi)^3} \left[\bar{a}(\omega, \vec{p})(i\omega + \rho p^2) a(\omega, \vec{p}) + \frac{1}{2}\varphi(\omega, \vec{p}) \left(r + a\frac{|\omega|}{p} + bp^2 \right) \varphi(-\omega, -\vec{p}) \right]$$
$$+ \frac{m\lambda}{2} \int \prod_{l=1}^{2} \frac{d\omega_l}{2\pi} \frac{d^3 p_l}{(2\pi)^3} (\vec{p}_1 \cdot \vec{p}_2) \bar{a}(\omega_1, \vec{p}_1) a(\omega_2, \vec{p}_2) \varphi(\omega_1 - \omega_2, \vec{p}_1 - \vec{p}_2) \quad (31)$$

where

$$\rho = mJ_r \quad (32)$$

is the spin stiffness constant.

Let us rewrite the spin vector (equation (8)) in terms of the vector $\vec{M}$ (equation (28)) and use the Holstein–Primakoff representation (equation (30)). Then the magnetization of the system per lattice site is given by the expression

$$\langle S_i^3 \rangle = m - \langle \bar{a}_i a_i \rangle. \quad (33)$$

The equation for the critical temperature $\langle S_i^3 \rangle = 0$ is

$$m = \langle \bar{a}_i a_i \rangle. \quad (34)$$

Equation (34) follows from the exact representation for the third component of the spin ((8), (30) and (33)). Hence it takes the magnon fluctuations into account exactly. The equation allows us to account for the magnon–magnon and magnon–paramagnon interactions, thus going beyond Moriya's theory.

In the spin-wave approximation and for magnon dispersion in the form $\epsilon_a(p) = \rho p^2$ one obtains for the Curie temperature

$$T_c = \kappa m^{2/3} \rho(m) \quad (35)$$

where the constant κ can be written in terms of gamma $\Gamma(z)$ and Riemann $\zeta(z, q)$ functions $\kappa = (\Gamma(\frac{3}{2})\zeta(\frac{3}{2}, 1)/4\pi^2)^{-2/3}$. In the spin-wave approximation the spin stiffness constant is given by equation (32). Hence, when the system approaches the quantum critical point ($m \to 0$), the critical temperature scales with magnetization like $T_c \sim m^{5/3}$.

One can improve the equation for the Curie temperature replacing the zero-temperature dimensionless magnetization m by the finite-temperature solution $m(T)$ of the mean-field equation (13):

$$T_c = \kappa m^{2/3}(T_c)\rho(m(T_c)). \quad (36)$$

For conventional weak ferromagnets $m(T_c) \sim m$ and equation (35) is an appropriate expression for Curie temperature. But for high T_c weak ferromagnets the correct equation for the critical temperature is equation (36).

In the spin-wave approximation the transverse components of the spin fields are proportional to the magnon fields

$$S_i^+(\tau) = \sqrt{2m}a_i(\tau), \qquad S_i^-(\tau) = \sqrt{2m}\bar{a}_i(\tau) \quad (37)$$

and the field $\varphi_i(\tau)$ is exactly the paramagnon (longitudinal spin fluctuation)

$$S_i^3(\tau) - \langle S_i^3 \rangle = \varphi_i(\tau). \quad (38)$$

Hence, in the Gaussian approximation, spin–spin correlation functions have the form

$$D^{tr}(\omega, \vec{p}) = \frac{2m}{\mathrm{i}\omega + \rho p^2}, \qquad D^{long}(\omega, \vec{p}) = \frac{1}{r + a|\omega|/p + bp^2} \tag{39}$$

where the longitudinal magnetic susceptibility is

$$\chi = D^{long}(0, 0) = \frac{1}{r}. \tag{40}$$

3. Scaling behaviour

Scaling arguments, based on Hertz's renormalization group technique allow us to obtain a relation between the spin stiffness and the parameter r–the inverse longitudinal magnetic susceptibility.

To begin with, I introduce a cut-off and redefine the momenta introducing a dimensionless one. RG construction starts with a definition of 'soft' and 'fast' modes:

$$\begin{aligned} &\Psi(\omega, \vec{p}) = \Psi_<(\omega, \vec{p}) + \Psi_>(\omega, \vec{p}) \\ &\Psi_<(\omega, \vec{p}) = \Psi(\omega, \vec{p}) \qquad \text{for } |p| < \mathrm{e}^{-l} \\ &\Psi_>(\omega, \vec{p}) = \Psi(\omega, \vec{p}) \qquad \text{for } \mathrm{e}^{-l} \leqslant |p| \leqslant 1 \end{aligned} \tag{41}$$

where $\Psi(\omega, \vec{p})$ stands for magnons, $a(\omega, \vec{p}), \bar{a}(\omega, \vec{p})$ and paramagnon $\varphi(\omega, \vec{p})$ fields. The action equation (31) depends on the 'slow' and the 'fast' modes. In the tree approximation one keeps only the 'slow' modes

$$S^{tree}(\bar{a}, a, \varphi) = S(\bar{a}_<, a_<, \varphi_<). \tag{42}$$

The next step is to change the variables, letting

$$\vec{p}\,' = \mathrm{e}^{l}\vec{p}, \qquad \omega' = \mathrm{e}^{zl}\omega. \tag{43}$$

The new momenta runs over the whole interval $0 \leqslant |p'| \leqslant 1$, and in terms of the new variables the action has the form

$$\begin{aligned} S^{tree} = {}& \mathrm{e}^{-(z+3)l} \int \frac{\mathrm{d}\omega'}{2\pi} \frac{\mathrm{d}^3 p'}{(2\pi)^3} \Big[\bar{a}_<(\mathrm{e}^{-zl}\omega', \mathrm{e}^{-l}\vec{p}\,')(i\mathrm{e}^{-zl}\omega' + mJ_r\mathrm{e}^{-2l}p'^2)a_<(\mathrm{e}^{-zl}\omega', \mathrm{e}^{-l}\vec{p}\,') \\ & + \frac{1}{2}\varphi_<(\mathrm{e}^{-zl}\omega', \mathrm{e}^{-l}\vec{p}\,')\left(r + \mathrm{e}^{(-z+1)l}a\frac{|\omega'|}{p'} + \mathrm{e}^{-2l}bp'^2\right)\varphi_<(-\mathrm{e}^{-zl}\omega', -\mathrm{e}^{-l}\vec{p}\,')\Big] \\ & + \Lambda\mathrm{e}^{-2(z+4)} \int \prod_{l=1}^{2} \frac{\mathrm{d}\omega'_l}{2\pi} \frac{\mathrm{d}^3 p'_l}{(2\pi)^3} (\vec{p}\,'_1 \cdot \vec{p}\,'_2)\bar{a}_<(\omega'_1, \vec{p}\,'_1)a_<(\omega'_2, \vec{p}\,'_2) \\ & \times \varphi_<(\omega'_1 - \omega'_2, \vec{p}\,'_1 - \vec{p}\,'_2), \end{aligned} \tag{44}$$

where I have used the short notation for the magnon–paramagnon coupling constant $\Lambda = m\lambda/2$.

It is apparent that, if one chooses $z = 3$, the coefficient of $a\frac{|\omega'|}{p'}$, in the paramagnon quadratic term, is the same as the coefficient of bp'^2. Then rescaling the paramagnon φ, one can make the total coefficient of both of them unity:

$$\varphi'(\omega', \vec{p}\,') = \mathrm{e}^{-z+5/2l}\varphi_<(\mathrm{e}^{-zl}\omega', \mathrm{e}^{-l}\vec{p}\,'). \tag{45}$$

To complete the RG transformation, one has to rescale the magnon fields too, to make the coefficient of $\mathrm{i}\omega'$ unity:

$$a_<(\omega', \vec{p}\,') = \mathrm{e}^{-2z+3/2}a_<(\mathrm{e}^{-zl}\omega', \mathrm{e}^{-l}\vec{p}\,'), \qquad \bar{a}_<(\omega', \vec{p}\,') = \mathrm{e}^{-2z+3/2}\bar{a}_<(\mathrm{e}^{-zl}\omega', \mathrm{e}^{-l}\vec{p}\,'). \tag{46}$$

Then, for the transformed action one obtains

$$S'(\bar{a}', a', \varphi') = \int \frac{d\omega'}{2\pi} \frac{d^3 p'}{(2\pi)^3} \left[\bar{a}'(\omega', \vec{p}')(i\omega' + \rho' p'^2) a'(\omega', \vec{p}') + \frac{1}{2}\varphi'(\omega', \vec{p}') \right.$$
$$\left. \times \left(r' + a\frac{|\omega'|}{p'} + bp'^2 \right) \varphi'(-\omega', -\vec{p}',) \right] + \Lambda' \int \prod_{l=1}^{2} \frac{d\omega'_l}{2\pi} \frac{d^3 p'_l}{(2\pi)^3}$$
$$\times (\vec{p}'_1 \cdot \vec{p}'_2) \bar{a}'(\omega'_1, \vec{p}'_1) a'(\omega'_2, \vec{p}'_2) \varphi'(\omega'_1 - \omega'_2, \vec{p}'_1 - \vec{p}'_2) \tag{47}$$

where r', ρ' and Λ' are the transformed parameters

$$\begin{aligned} r' &= r(l) = r\mathrm{e}^{2l} \\ \rho' &= \rho(l) = \rho \mathrm{e}^{(z-2)l} = \rho \mathrm{e}^{l} \\ \Lambda' &= \Lambda(l) = \Lambda \mathrm{e}^{z-5/2l} = \Lambda \mathrm{e}^{-l}. \end{aligned} \tag{48}$$

Excluding the scaling parameter l one obtains the relations (RG invariants)

$$\frac{r'}{\rho'^2} = \frac{r}{\rho^2}, \qquad r'\Lambda'^2 = r\Lambda^2. \tag{49}$$

The bare parameter r scales with magnetization like $r = r_0 m^2$, and the bare spin stiffness constant is proportional to the magnetization (see equation (31)). Hence, for weak ferromagnets, one obtains

$$r(m) \sim \rho^2(m). \tag{50}$$

The relation equation (50) is a consequence of the fact that the quantum critical exponent is equal to three ($z = 3$). It holds even if the magnon–magnon interaction is introduced. Calculating the magnon contribution to the spin stiffness, one can use it to obtain the magnon scattering corrections to the inverse longitudinal magnetic susceptibility r. For weak itinerant ferromagnets r is small and, as follows from the second of equations (49), the magnon–paramagnon coupling constant Λ is large. Hence, near the quantum phase transition, the magnon–paramagnon interaction is strong and cannot be treated perturbatively.

Making use of equation (50) one can obtain that

$$T_c \sim m^{2/3} T_s \tag{51}$$

where $T_s = \text{constant} \times \sqrt{r} \sim m$ is Stoner's expression for the critical temperature. For weak ferromagnets $m < \frac{1}{2}$, and T_c is well below Stoner's critical temperature, as it should be. The Curie temperature obtained within Moriya's theory scales with magnetization like $\sim m^{3/2}$ [18].

The result indicates that the spin-m Heisenberg model coupled to a paramagnon provides a good description of the itinerant ferromagnets.

4. Conclusions

The advantage of the present approach is the explicit separation of the spin fluctuations and the charge carriers realized by means of the Schwinger-bosons–slave-fermions representation of operators. One can represent the Hamiltonian equation (10) as a sum of two terms $H = H^{\text{naked}} + H'$, where H^{naked} depends on charge carriers h_i and d_i only, while H' depends on spinon fluctuations $f_{i\sigma}$ and describes spinon–fermion interaction. The investigation of itinerant ferromagnets can be achieved in two steps. Within the 'naked' model, with Hamiltonian H^{naked}, one can obtain an expression for the magnetization $m = \frac{1}{2}(1 - \langle \bar{h}_i h_i + \bar{d}_i d_i \rangle$, which is exact at zero temperature. Equation (36) accounts for the spin fluctuations exactly. One can see from the Hamiltonian H' that the composed fields $h_i d_j - h_j d_i$ are associated with the transverse spin fluctuations, while the fields $\bar{h}_i h_i + \bar{d}_i d_i$ are associated with the longitudinal one. The

Green functions of these fields give us the parameters of the effective theory equation (27). The next step is to study the magnon–paramagnon effective theory itself.

I use a mean-field approximation to study the 'naked' theory and to obtain the coefficients in the effective theory. To this end the direct Heisenberg exchange interaction which favours ferromagnetism in an obvious way is necessary. The Hubbard term is diagonalized, which enables us to account for the on-site Coulomb repulsion exactly. As a result, I obtain that the vertices in the effective functional exist in the limit of zero frequencies and wavenumber, and that the constants are well defined for all values of the magnetization.

For a theory with $J = 0$, one has to go beyond the mean-field theory. But, even in that case, the 'naked' theory is preferable to the original Hubbard model. Coulomb repulsion U is the largest energy scale in the problem, and it is desirable to diagonalize this term. It is not adequate to drop it, as in [11], or to treat it perturbatively, as in [13, 14]. Going beyond the mean-field theory, one obtains the same effective magnon–paramagnon theory with different expressions for the coefficients and different conditions for the ferromagnetic instability.

The effective model differs from the models discussed in [11, 13, 14] in many ways. It describes in a unified way both the ordered and disordered phases of the system. Altering the parameters, it interpolates between the Heisenberg theory of localized spins and Hertz's theory of nearly ferromagnetic metals. In the ferromagnetic phase the important spin fluctuations are transversal magnon fluctuations and longitudinal paramagnon fluctuations. In the thermal paramagnetic phase (above the Curie temperature) the spectrum consists of spin-singlet fluctuations of paramagnon type and spin-$\frac{1}{2}$ spinon fluctuations. Well above the critical temperature the spinon has a large gap, and the physics of ferromagnetic metals is dominated by the paramagnon fluctuations. But just above T_c the spinon's gap approaches zero [24] and the contribution of the spin-$\frac{1}{2}$ fluctuations is essential. The very strong spinon–paramagnon interaction near the quantum phase transition breaks the 'spin-liquid' picture, which in turn leads to the formation of a non-Fermi liquid, in accordance with experimental observation [16]. Crossing the quantum critical point ($m = 0$), the spinon excitations disappear from the spectrum and only the paramagnon survives in the quantum paramagnetic phase.

The effective model equations (27) and (31) enable us to estimate to what extend the Green functions calculations [4], and the Moriya–Kawabata approximation [6] are applicable. Within these approaches, the spin stiffness constant is proportional to the magnetization. The same result follows from the effective model equation (31) where the nonlinear magnon–magnon interaction is not considered. Moriya's theory is a very successful description of many ferromagnetic systems for which the spin-wave approximation gives an adequate account of the spin fluctuations. On the other hand, it follows from the spin-wave expansion that the magnon vertices are of order $(\frac{1}{m})^n$. Hence, on the verge of ferromagnetism they are relevant, and one has to go beyond Moriya's theory. One way to do this is to use a renormalization group approach in the spin-wave theory, which allows for the analysis of systems with small spins [25]. The magnon–magnon interaction changes the small magnetization asymptote of the spin stiffness constant $\rho = m^{1+\alpha}\rho_0$, where $\alpha > 0$.

Acknowledgments

The author would like to thank K Bedell and K Blagoev for valuable discussions. This research is supported by the USA National Science Foundation Grant-DINT no 9876873.

References

[1] Abrikosov A A and Dzyloshinskii I E 1959 *Zh. Eksp. Teor. Fiz.* **35** 771 (Engl. transl. 1959 *Sov. Phys.–JETP.* **8** 535)

[2] Ramakrishnan T V 1974 *Solid State Commun.* **14** 449

Ramakrishnan T V 1974 *Phys. Rev.* B **10** 4014
[3] Kawabata A 1974 *J. Phys. F: Met. Phys.* **4** 1477
[4] Dzyaloshinskii I E and Kondratenko P S 1974 *Zh. Eksp. Teor. Fiz.* **70** 1987 (Engl. transl. 1975 *Sov. Phys.–JETP* **43** 1036)
[5] Murata K and Doniach S 1972 *Phys. Rev. Lett.* **29** 285
[6] Moriya T and Kawabata A 1973 *J. Phys. Soc. Japan* **34** 639
[7] The works are summarized in
Moriya T 1985 *Spin Fluctuations in Itinerant Electron Magnetism* (Berlin: Springer)
[8] Lonzarich G and Taillefer L 1985 *J. Phys. C: Solid State Phys.* **18** 4339
[9] Metzner W and Vollhardt D 1989 *Phys. Rev. Lett.* **62** 324
[10] Wahle J, Blümer N, Schlipf J, Held K and Vollhardt D 1998 *Phys. Rev.* B **58** 12749
[11] Hertz J 1976 *Phys. Rev.* B **14** 1165
[12] Millis A J 1993 *Phys. Rev.* B **48** 7183
[13] Vojta T, Belitz D, Narayanan R and Kirkpatrick T R 1996 *Europhys. Lett.* **36** 191
[14] Vojta T, Belitz D, Narayanan R and Kirkpatrick T R 1997 *Z. Phys.* B **103** 451
[15] Pfleiderer C, McMullan G J, Julian S R and Lonzarich G G 1997 *Phys. Rev.* B **55** 8330
[16] Pfleiderer C, Julian S R and Lonzarich G G 2001 *Nature* **414** 427
[17] Brinkman W F and Engelsberg S 1968 *Phys. Rev.* **169** 417
[18] Gumbs G and Griffin A 1976 *Phys. Rev.* B **13** 5054
[19] Hubbard J 1963 *Proc. R. Soc.* A **276** 238
Hubbard J 1964 *Proc. R. Soc.* A **277** 237
[20] Hirsh J E 1989 *Phys. Rev.* B **40** 2354
Hirsh J E 1989 *Phys. Rev.* B **40** 9061
[21] Vollhard D, Blümer N, Held K, Kollar M, Schlipf J and Ulmke M 1997 *Z. Phys.* B **103** 283
[22] Negele J W and Orlando H 1988 *Quantum Many-Particle Systems* (Reading, MA: Addison-Wesley)
[23] Schmeltzer D 1991 *Phys. Rev.* B **43** 8650
[24] Arovas D P and Auerbach A 1988 *Phys. Rev.* B **38** 1316
[25] Karchev N 1997 *Phys. Rev.* B **55** 6372